THE RANDOM CODED MODULATION: PERFORMANCE AND EUCLIDEAN DISTANCE EVALUATION

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1 Bounds on the error probability

This paper is intended to apply Shannon's random coding argument [1] to coded modulation [2] to derive bounds on the error probability. We show that
\[ P_e(M) \leq \sum_{d \geq 0} \sum_{\{|d|=d\}} N_d \epsilon^d, \]
where \( P_e(M) \) is the error probability of the M-dimensional (M-D) constellation with \( M \) equally likely signal points on an additive white Gaussian noise (AWGN) channel with receive signal-to-noise ratio \( S/N \). \( N_d \) is the number of points at a squared Euclidean distance of \( d \) from a given point in the coded or uncoded constellation.

Let us now consider the M-D uncoded constellation result from the Cartesian product of a constant-D constellation (of size \( q \) and minimum distance \( d_0 \)) with itself N times. We have \( \epsilon = P_e(M) \). To prove this, we use the error probability per 2-D symbol. From (2), we have \( N_d \) is the number of points at a squared Euclidean distance of \( d \) from a given point in the 2-D uncoded constellation.

Let us denote the distance spectrum of the uncoded and random coded modulations be \( D^u(M) = \sum_{d \geq 0} N_d \epsilon^d \) and \( D^r(M) = \sum_{d \geq 0} N_d \epsilon^d \), respectively. Assuming that the cross-dimensional random coding is the same as the uncoded unidimensional signal set results in \( \epsilon = \epsilon^d \). \( N_d^r(M) = q^{d/2} N_d(M) = \sum_{d \geq 0} n_d(M) \epsilon^d \) for \( d \geq 0 \). We have
\[ \epsilon = \sum_{d \geq 0} n_d(M) \epsilon^d. \]

\[ n_d(M) = \sum_{d \geq 0} n_d(M) \epsilon^d. \]

Asymptotic Behavior of Very Long Codes

Given an arbitrary \( \epsilon > 0 \), if we pick at random a particular coded (individually encoded) signal set with length \( N \), there is a good chance that the average squared Euclidean distance of the signal set will be larger than \( d_0 \). This is due to the statistical behavior of the signal set as \( N \) increases. As \( N \) becomes large, the average squared Euclidean distance tends to infinity. For \( \epsilon < 0.1 \), about 40 coded signal sets are needed to ensure that the average squared Euclidean distance of the signal set will be larger than \( d_0 \). If \( \epsilon = 0.1 \), about 100 coded signal sets are needed.

Concluding Remarks

We applied Shannon's random coding argument to coded modulation. Asymptotically, we found that the normalized squared Euclidean distance exhibits a limiting phenomenon. This means that the normalized distance does not increase with the number of signal sets as \( N \) becomes very large. However, this is not true for very small \( N \) values.

References


\[ \sum_{d \geq 0} n_d(M) \epsilon^d = \sum_{d \geq 0} n_d(M) \epsilon^d. \]