Wavelet generation circuit for UWB impulse radio applications

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A simple circuit for generating wavelets required in an ultra-wideband (UWB) impulse radio receiver is presented. The second derivative \( G_2(t) \) of the Gaussian function is approximated by combining multiple hyperbolic tangent (tanh) functions. Simulations show that a good approximation of this waveform is possible using integer coefficients. A bipolar transistor circuit implementation is presented and its simulation results shown.

Introduction: Impulse radio uses short-duration pulses as data carrier, resulting in a spreading of the radio signal over a wide frequency band [1]. Suppose that the transmitter generates a Gaussian pulse \( G_0(t) \) at time \( t = 0 \) of temporal width \( \tau_m \) described by

\[
V_{R X}(t) = G_0(t) = \exp\left(-2\pi \left( \frac{t}{\tau_m} \right)^2 \right) \tag{1}
\]

The ideal voltage response \( V_{RX}(t) \) of the receiver antenna in the absence of multipath is a delayed and attenuated version of the second derivative \( G_2(t) \) of this pulse [2] described by

\[
V_{RX}(t) = k G_2(t - \tau_d) = k(1 - 4\pi^2 x^2) \exp(-2\pi^2 x^2) \quad \text{with} \quad x = t - \frac{\tau_d}{\tau_m} \tag{2}
\]

In a correlation receiver, detection of the received signal \( V_{RX}(t) \) is performed by multiplying the incoming signal with a replica of the \( G_2 \) function and integrating the result over an appropriate time interval. Generation of the \( G_2 \) function is therefore essential in correlation receivers. What kind of circuit would be appropriate to generate this particular waveform?

It is well-known that the output current \( I_{out} \) of a bipolar differential pair with tail current \( I_T \) and input voltage \( V_{in} \) is equal to

\[
I_{out} = I_T \tanh \left( \frac{V_{in}}{2V_T} \right) \quad \text{with} \quad V_T = \frac{kT}{q} \tag{3}
\]

This nonlinear transfer function is often exploited for waveform shaping; the triangle-to-sine wave conversion being probably the most popular one [3]. In this Letter, we describe the generation of the \( G_2 \) signal out of a sawtooth-like input signal \( x(t) \) using a multiple hyperbolic tangent approximation technique. As a result, the circuit implementation requires only differential pairs and current sources and is well suited for IC integration.

Approximation technique: The second derivative \( G_2(x) \) of the Gaussian function is approximated by the function \( H(x) \) composed of the weighted sum of four hyperbolic tangent functions

\[
H(x) = -0.5 \tanh(x + 2) + \tanh(x + 1) - \tanh(x - 1) + 0.5 \tanh(x - 2) \tag{4}
\]

Fig. 1 shows the hyperbolic tangent function \( \tanh(x) \), the function \( G_2(x) \), its approximation \( H(x) \) and the difference between the two. \( H(x) \) has been normalised with respect to its maximum. The error term \( (H - G_2) \) has an RMS value of 2%, ensuring good correlation properties of the functions \( G_2(x) \) and \( H(x) \). This approximation method can be used to approximate almost any wavelet function, e.g. the Gaussian doublet. The number of terms required for the approximation \( H(x) \) is equal to the number of slopes of the waveform to be approximated.

Circuit realisation: By combining the output signals of four bipolar differential pairs, the function \( H \) is realised at circuit level. Fig. 2 shows the resulting schematic diagram. Scaling of the individual tail currents and offset voltages of the differential pairs implement the coefficients of (4). Negative coefficients are obtained by cross-coupling the output connections.

Results: Applying a linear ramp as input voltage \( V_{in} \) to this circuit yields the approximation of the second derivative of the Gaussian function as output signal. Fig. 3 shows the results of a SPICE transient simulation. The input signal is a pulse with unequal rise and fall times. The shorter fall time of the input signal results in a narrower output pulse. The transistor models used represent a bipolar technology with transition frequency \( f_T = 10 \) GHz. It can be seen that narrower pulses tend to become asymmetric due to the circuit’s bandwidth limitations.

![Fig. 1 Approximation H of G2 function using multiple tanh functions](image1.png)

![Fig. 2 Circuit implementation of G2 approximation circuit](image2.png)

![Fig. 3 Spice simulation results of G2 approximation circuit for input signal with different rise and fall times (2 and 1 ns)](image3.png)
Conclusions: A straightforward bipolar circuit generating the second derivative of the Gaussian pulse from a sawtooth-like input signal has been presented. A good approximation showing 2% RMS error with the second derivative of the Gaussian function has been shown possible using four differential pairs and four current sources with scaling factors of 1 and 2. The output pulse width can be adjusted by changing the rise and fall time of the input signal.

References